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AN INVESTIGATION OF THE VELOCITY OF SOUND IN AMMONIA

BY

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THESIS

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY

SUPERVISION BY Floyd Dunn

ENTITLED An Investigation of the Velocity of Sound in Ammonia

BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR

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M440

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I

Introduction

In general, the principal motives for experimental research are either to corroborate theoretical investigations or to conduct more accurate measurements of physical phenomena. The work presented in this paper involves both of these motives. Specifically, measurements have been made of the velocity of sound in ammonia in the region which has heretofore been thought to be a region of anomalous dispersion. These measurements show that the velocity decreases slightly with increasing frequency. The theory of sound propagation in gases which includes losses due to shear viscosity, heat conductivity, and thermal relaxation does not predict such a variation.

The literature shows that a relatively small amount of work has been done in measuring the velocity of sound in ammonia. In the two cases discussed in the text of this paper, both workers measured the wavelength of the sound wave at various pressures above and below atmospheric pressure. Their results show that the wavelength varies inversely with the pressure. If it is assumed that the velocity is a function of $(\frac{\omega}{P})$, then their results show that the velocity varies directly with $(\frac{\omega}{P})$ and, hence, dispersion. In this paper it is shown that these results can be computed by using an exact expression for velocity of propagation in a gas together with the equation of state for ammonia. Thus, the two works discussed do not conclusively demonstrate the existence of dispersion in the region of investigation, since the results are consequences of pressure variation and not of frequency variation.

In the experiments on ammonia presented here, a steady state method was used to measure the wavelength. A double-crystal acoustic interferometer, which is described from a theoretical point of view by W. J. Fry[1*], was the principle device used. This interferometer was designed in such a manner as to make the displacement of the movable crystal almost entirely independent of the screw mechanism. The variation in frequency was accomplished by using different sets of crystals which ranged from 191 KC to 1210 KC.

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*Superscripts refer to numbers in the bibliography.

II

Discussion of the Velocity of Sound in Ammonia

A one dimensional discussion of sound propagation in gases and complete derivations of the equations expressing velocity and absorption are to be found in the works of several authors[5,6,8]. These authors take into consideration losses due to heat conductivity, shearing viscosity, and thermal relaxation. The effect of thermal conductivity of the gas gives rise to some transfer of thermal energy within the sound wave between successive warm and cool half cycles. The viscous losses in a plane wave arise when a motion in which one face of a cubic volume element is displaced while the perpendicular faces remain stationary. Concerning heat conductivity and viscosity, it is found that the sound velocity is given by*

$$1. \qquad V = \left[\gamma \left(\frac{RT}{M} \right) \right]^{\frac{1}{2}},$$

where V is the velocity, γ is the ratio of heat capacities, T is Kelvin temperature, and R is the gas constant. Eq. 1 is the usual adiabatic velocity, i.e., these losses do not affect the velocity of propagation. Two approximations are made in deriving eq. 1. First it is assumed that the wavelength of sound is very much greater than the molecular mean free path. For ammonia the mean free path at 75cm. Hg and 20° C is 6.6×10^{-6} cm.

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*The appendix contains a list of the symbols used.

while the shortest wavelength measured in these experiments was 3.5 X 10^{-1} cm. The second approximation assumes that the wave vector, k, which is defined by

$$p = p_0 e^{-k_2 x} e^{j(\omega t - k_1 x)} ,$$

$$k = k_1 - j k_2 ,$$

where p is the local excess pressure in the gas, ω is the angular frequency, and x the direction of propagation of the plane wave of sound, can be approximated by

$$k^2 = k_1^2 - k_2^2 - 2jk_1k_2 \cong k_1^2 - 2jk_1k_2$$

(k_1 determines the wavelength and k_2 determines the pressure attenuation.) This approximation can be made for nearly all gases. Thus it is seen that the two approximations made in deriving eq.1 are justified for ammonia.

Losses due to thermal relaxation arise from incomplete establishment of thermal equilibrium in a system. When all parts of a system are not at the same temperature there arises a loss due to energy dissipation. The energy dissipation is most pronounced when the period of the heating and cooling cycle is comparable with the time required for energy transfer between the internal and external parts of the system. If the molecule is considered to be divided into two thermodynamic systems, an external one and an internal one, the sound velocity is found to be

$$2. \qquad V = \left[\left(\frac{RT}{M}\right)\left(\frac{C_p^{0\,2} + \omega^2\tau^2 C_p^{\infty\,2}}{C_p^0 C_v^0 + \omega^2\tau^2 C_v^\infty C_p^\infty}\right)\right]^{\frac{1}{2}}$$

where τ is the relaxation time constant and the C's are the molar heat capacities for constant pressure and volume at high and low frequencies. The velocity is seen to be a function of frequency. Examination of eq.2 shows that at low frequencies the velocity is that given by eq.1. For very high frequencies, where there is no time for an energy transfer between the internal and external parts of the system in the time of a cycle, the specific heats are different and the velocity is given by

$$V = \left[\left(\frac{RT}{M}\right)\left(\frac{C_p^{\infty}}{C_v^{\infty}}\right)\right]^{\frac{1}{2}} .$$

According to the kinetic theory of gases, as the pressure of the gas increases the number of collisions per unit time increases. If the mean number of collisions necessary to effect a change of state is fixed, then τ will be inversely proportional to the pressure and may be written as

$$\tau = \frac{\gamma}{P} ,$$

where γ is a constant. Substituting this into eq.2 gives

3. $$V = \left[\left(\frac{RT}{M}\right)\left(\frac{C_p^{o2} + \gamma^2\left(\frac{\omega}{P}\right)^2 C_p^{\infty 2}}{C_p^{o} C_v^{o} + \gamma^2\left(\frac{\omega}{P}\right)^2 C_p^{\infty} C_v^{o}}\right)\right]^{\frac{1}{2}} .$$

The velocity is now a function of $\frac{\omega}{P}$.

The work done by Steil[7] and Welkowitz[8] was based essentially on eq. 3. Steil used a Pierce interferometer and Welkowitz used a double-crystal interferometer. Both workers* varied the pressure while keeping

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*Actually Steil used two different frequencies to investigate the region of $\log\frac{\omega}{P}$ from 3.4 to 4.2.

the frequency constant and were able to measure a change in the wavelength of sound as a function of pressure. The wavelength varied inversely with the pressure. Welkowitz's data is shown in Fig.1.

It will now be shown that variation in the velocity of sound as a function of pressure, found by Steil and Welkowitz, is to be expected. From the solution of the differential equation for the propagation of sound in a gas one obtains[2]

$$4. \qquad V = \left(\frac{E_A}{\rho}\right)^{\frac{1}{2}} = \left(\frac{C_p}{C_v}\,\frac{E_T}{\rho}\right)^{\frac{1}{2}} ,$$

where

$$5. \qquad E_T = -v\left(\frac{\partial P}{\partial v}\right)_T ,$$

where ρ is the density, v the specific volume, and E_A and E_T, respectively the adiabatic and isothermal elasticities. This expression is exact, provided the amplitude of the sound is not too great. This condition is fulfilled in most experiments and in particular in the work presented in this paper and in Welkowitz's work.

The equation of state for ammonia[3,4] is

$$6. \qquad P = \frac{RT}{v-\delta} - \frac{a}{(v-\ell)^2} ,$$

where

$$\log_{10}\delta = 0.98130 - \frac{3.08}{v} ,$$

$$a = 34610.1 ,$$

$$\ell = -1.173 .$$

The constants of this equation were experimentally determined and thermodynamic tables were made-up under the direction of F. G. Keyes.[4]

If eq.6 is substituted into eq.4, one obtains for the velocity

$$7. \quad V^2 = \left[1 + \frac{c_p - c_v}{c_v}\right]\left[-v^2\left(\frac{\partial P}{\partial v}\right)_T\right],$$

where

$$8. \quad c_p - c_v = \frac{R}{\left(1 - \frac{\alpha\delta}{v^2}\right) - \frac{2a}{RT}\frac{(v-\delta)^2}{(v-l)^3}},$$

and

$$9. \quad \left(\frac{\partial P}{\partial v}\right)_T = -RT\frac{\left(1 - \frac{\alpha\delta}{v^2}\right)}{(v-\delta)^2} + \frac{2a}{(v-l)^3}.$$

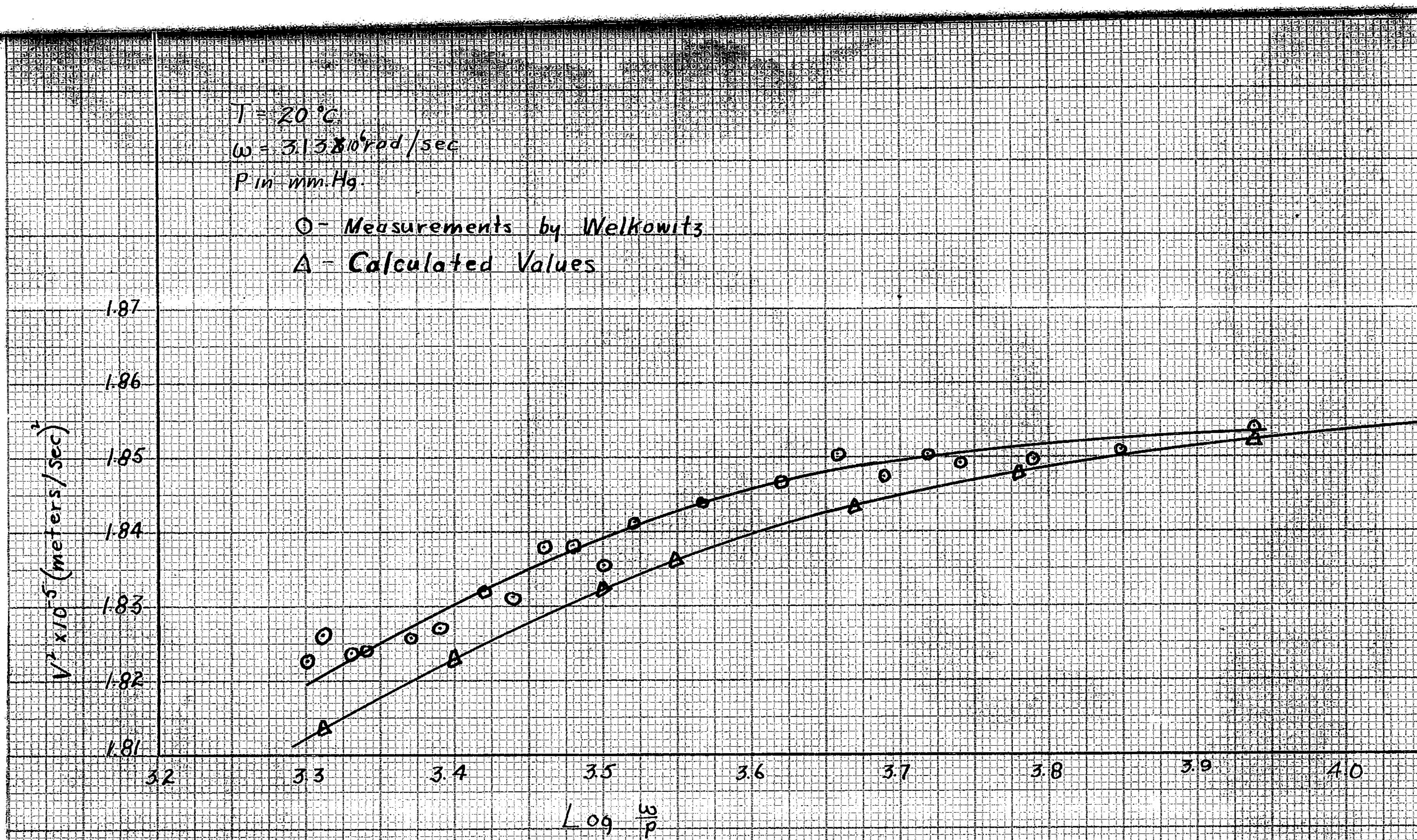
The velocity of sound in ammonia can now be computed as a function of pressure by using eqs.7,8, and 9 in conjunction with the data published by Keyes and Brownlee.[4] When this is done the following values are obtained for the velocity.

T(°C)	P(mm.Hg)	$\log\frac{\omega}{P}$	$v^2 \times 10^{-5}$(meters/sec)
20	5172	2.78	1.702
	2482	3.10	1.784
	1552	3.31	1.813
	1241	3.40	1.822
	983	3.50	1.832
	879	3.55	1.836
	672	3.67	1.842
	517	3.78	1.847
	362	3.94	1.851
	259	4.08	1.855

(ω was taken as 3.133×10^6 rad/sec in order that the above values may be compared with Welkowitz's experimental data.)

These values are plotted in Fig.1 together with Welkowitz's data. The agreement between the two is seen to be quite good. It must be considered that Welkowitz's data was taken at temperatures ranging from 24.9°C to 30.3°C and then corrected to 20°C. by the perfect gas relationship, i.e., eq.1. Eqs.6 and 7 show that ammonia is a comparatively poor perfect gas and that corrections made over a range of 5°C to 10°C by eq.1 are probably not justified.

It is interesting to note that if the values computed above are subtracted from Welkowitz's data, the resulting velocity tends to decrease slightly with $\frac{\omega}{p}$. This is the same phenomenon that was found by varying the frequency and is presented in a latter part of this paper. The same thing is found to be true with respect to Steil's work.



The Velocity of Sound in Ammonia
as a Function of Pressure
Figure 1

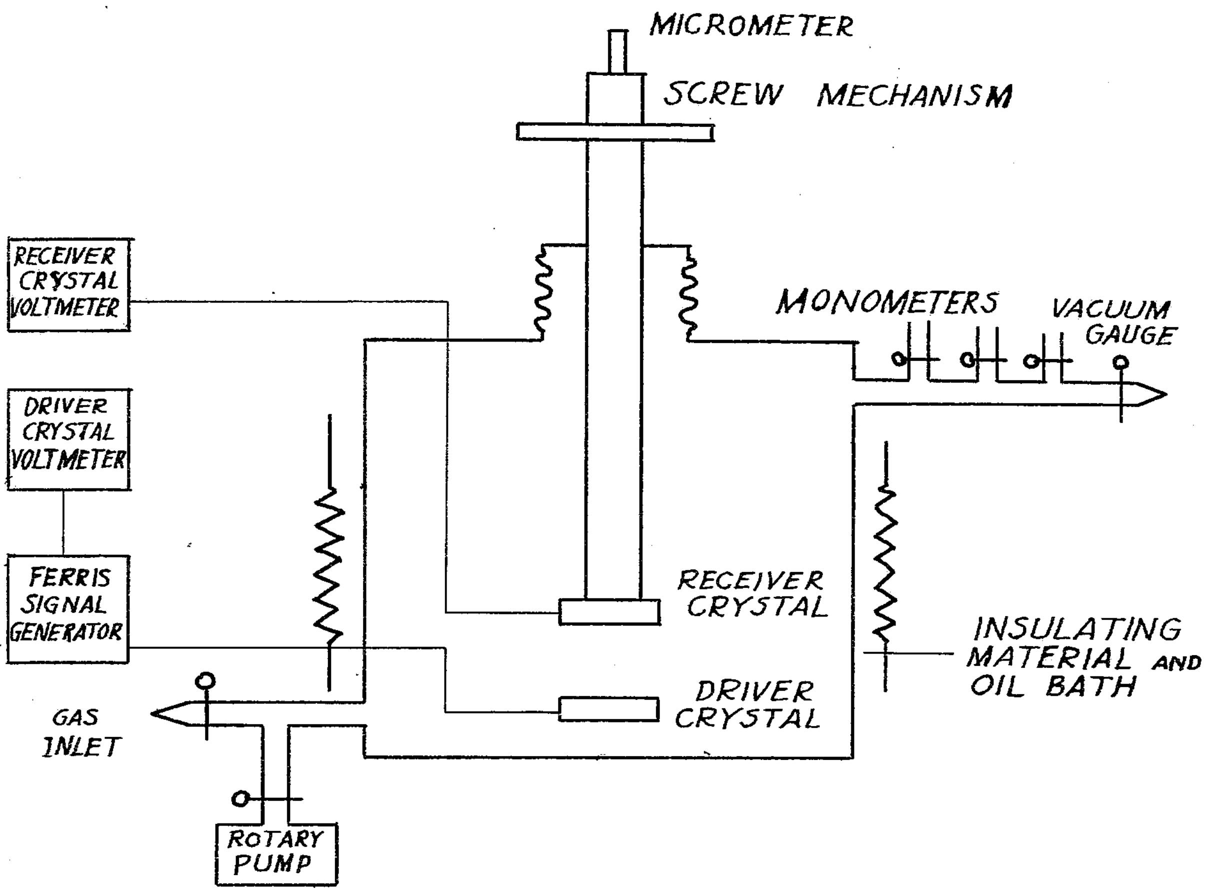
III

Description of Experimental Apparatus

The experimental apparatus is shown schematically in Fig.2. The principal device used was a double-crystal acoustic interferometer.

A Ferris signal generator, model 22-A, was used to drive the lower crystal. The input voltage to the driving crystal and the output voltage of the receiving crystal were measured with a Hewlitt-Packard vacuum tube voltmeter, model 400C. The position of the receiving crystal with respect to the driving crystal was adjusted by a screw mechanism. The screw mechanism was so arranged that the shaft holding the receiving crystal was guided by both a tightly fitted dove-tail bearing and a finely machined cylindrical bearing. This rendered the crystal motion as independent of the screw as was possible. A metal bellows was sealed to both the receiving crystal shaft and the top of the interferometer. This insured vacuum tightness of the interferometer while the shaft was being moved. The displacement of the shaft, and hence, the upper crystal, was measured with a micrometer which could be read directly to 0.01 mm.

The frequency of vibration of the crystals was measured with a U. S. Army Signal Corps frequency meter, model BC-221. The meter was checked with Station WWV and was capable of measuring frequency to 10 c.p.s. The gas pressure in the interferometer was measured with a mercury monometer. The heights of the mercury columns of the monometer were measured with a cathotometer which could be read directly to 0.05 mm. Atmospheric pressure was measured with a mercurial barometer which could be read



Equipment Not Shown
1. Frequency Meter
2. Cathetometer

Schematic Diagram of Double-Crystal Acoustic Interferometer and Associated Measuring Equipment

Figure 2.

directly to 0.10 mm. The interferometer was set in an oil-bath to reduce temperature fluctuations. The temperature of the oil-bath was measured with a thermometer which could be read directly to 0.2° C.

IV

The Experimental Method

From the theory of the double-crystal acoustic[1] interferometer, it is shown that the ratio of the receiver crystal voltage to the driver crystal voltage is

$$\left|\frac{E_R^B}{E_D^A}\right| = \frac{|C''|}{\left[\cosh^2\left(\alpha\frac{L}{\lambda}+b'\right)-\cos^2\left(2\pi\frac{L}{\lambda}+a'\right)\right]^{\frac{1}{2}}},$$

For constant driver crystal voltage, the receiver crystal voltage is seen to be a damped peaked curve for increasing L. The peaks occur for

$$\cos^2\left(2\pi\frac{L}{\lambda}+a'\right) \sim 1,$$

$$2\pi\frac{L}{\lambda}+a' \sim n\pi,$$

n an integer,

and it is seen that the distance between adjacent peaks is very nearly $\frac{\lambda}{2}$, a half-wave length. In the appendix a calculation is made which shows that the error in measuring the distance between 25 wavelengths is of the order of 0.004 wavelengths. Thus, the above approximations are justified.

In the development of the theory of the double-crystal acoustic interferometer several assumptions are made concerning the crystals. These assumptions are: the crystals are parallel, they are matched in frequency, and they are vibrating like pistons. The crystals were paralleled in three successive steps. First, they were brought very close together and the slit between them was viewed from two orthogonal directions. Second, a refinement of the parallelism was made by adjusting the voltage ratio and the standing wave ratio to a maximum. Third, the final refinement of the parallelism was made by minimizing the variation in the distance between successive standing wave peaks. The crystals were matched in frequency to one cycle by the supplier. At present there is no direct method available to determine the mode of vibration of the crystals. However, measurements were made of the velocity of sound in air and found to agree with accepted values. With these conditions satisfied, it was assumed that the interferometer was working according to theory.

The method of operation of the interferometer was as follows: The receiver crystal was moved with respect to the driver crystal and the position of a peak was located by the receiver crystal voltmeter. The position of the peak was then measured with the micrometer.

The data were taken in such a manner as to minimize errors introduced by the screw mechanism in locating a peak. Two different methods were used depending upon the frequency (i.e., wavelength) of a particular set of crystals. First, if the wavelength was such that a small integral number of revolutions of the screw equaled an integral number of half-wave lengths, then the data were taken for a fixed peak-interval. The peak-interval being determined by the number of peaks (or half-wavelengths) per revolution of the screw. Second, if the wavelength was such that a large number of revolutions of the screw equaled an integral number of half-wavelengths, then the data were taken for each peak in two regions of the movement of the upper crystal, near the lower crystal and far from the lower crystal.

Before taking data with any particular set of crystals, several precautions were taken. The system was pumped down to approximately 30 μ pressure and was then flushed with ammonia. After flushing, the system was again pumped down to 30 μ pressure. The system was then filled with ammonia to 1200 mm. pressure and was allowed approximately 30 minutes to come to equilibrium. The temperature of the oil-bath was brought as close to 26° C as was possible and approximately 3 hours was allowed for temperature equilibrium to take place. The signal generator was allowed approximately 6 hours warm-up time. After such a prolonged warm-up time it was found that the generator was sufficiently stable and no appreciable frequency drifting was noticed.

A chemical analysis of the ammonia used showed it to have a purity of 98.4%.

V

Data and Calculations

The data will be presented seperately for each set of crystals, i.e., for each frequency. The calculations of the velocity will follow each set of data. The calculations including temperature corrections will be collected at the end of this section.

The following is a list of the crystals used. The frequencies are approximate.

A. 191 KC	D. 344 KC
B. 245 KC	E. 429 KC
C. 308 KC	F. 609 KC
G. 1210 KC	

For crystal-set A, readings were taken for each peak from the 1st peak to the 7th peak and from the 11th peak to the 17th peak.

1.

Peak	Spacing (mm)
1	6.988
2	8.123
3	9.259
4	10.377
5	11.500
6	12.631
7	13.766
11	18.270
12	19.403
13	20.537
14	21.650
15	22.774
16	23.899
17	25.042

f = 191.45 KC

T = 25.4° C

p = 1200.0 mm

E_D^A = 0.95 volts

Subtracting the 1st peak from the 11th peak, 2nd from 12th, etc., and averaging gives

$$5\lambda = 11.2758 \text{ mm}$$

$$\lambda = 2.25516 \text{ mm}$$

$$V = f = 431.750 \text{ meters/sec}$$

2\.

Peak	Spacing (mm)
1	6.990
2	8.126
3	9.256
4	10.378
5	11.502
6	12.632
7	13.769
11	18.272
12	19.408
13	20.539
14	21.650
15	22.777
16	23.902
17	25.042

$f = 191.45$ KC

$T = 25.6^\circ$ C

$p = 1200.0$ mm

$E_D^A = 0.95$ volts

$$5\lambda = 11.2767 \text{ mm}$$

$$\lambda = 2.25534 \text{ mm}$$

$$V = 431.784 \text{ meters/sec}$$

3\.

Peak	Spacing (mm)
1	6.995
2	8.125
3	9.252
4	10.375
5	11.501
6	12.633
7	13.769
11	18.272
12	19.411
13	20.541
14	21.651

$f = 191.45$ KC

$T = 25.6^\circ$ C

$p = 1200.0$ mm

$E_D^A = 0.95$ volts

15	22.779
16	23.903
17	25.046

5λ = 11.2790 mm

λ = 22.5580 mm

V = 431.872 meters/sec

4.

Peak	Spacing (mm)
1	6.990
2	8.126
3	9.261
4	10.371
5	11.500
6	12.626
7	13.768
11	18.271
12	19.400
13	20.540
14	21.660
15	22.779
16	23.901
17	25.045

f = 191.45 KC

T = 25.6°C

p = 1200.0 mm

$\hat{E}_D$ = 0.95 volts

5λ = 11.2791 mm

λ = 2.25582 mm

V = 431.876 meters/sec

For crystal-set B, readings were taken for each peak from the 1st peak to the 8th peak and from the 16th peak to the 24th peak.

5.

Peak	Spacing (mm)
1	4.535
2	5.449
3	6.335
4	7.193
5	8.111
6	8.974
7	9.860
8	10.769
17	18.685
18	19.589

f = 244.73 KC

T = 26.3°C

p = 1200.0 mm

$\hat{E}_D$ = 0.90 volts

19	20.490
20	21.333
21	22.250
22	23.121
23	23.988
24	24.902

Subtracting the 1st peak from the 17th peak, etc., and averaging gives

$$8\lambda = 14.1415 \text{ mm}$$

$$\lambda = 1.76768 \text{ mm}$$

$$V = 432.604 \text{ meters/sec}$$

6.

Peak	Spacing (mm)
1	4.537
2	5.452
3	6.332
4	7.198
5	8.111
6	8.975
7	9.861
8	10.770
17	18.692
18	19.593
19	20.491
20	21.339
21	22.256
22	23.131
23	24.003
24	24.913

$f = 244.73$ KC

$T = 26.6^\circ$ C

$p = 1200.8$ mm

$E_0^A = 0.93$ volts

$$8\lambda = 14.1480 \text{ mm}$$

$$\lambda = 1.76850 \text{ mm}$$

$$V = 432.805 \text{ meters/sec}$$

7.

Peak	Spacing (mm)
1	4.539
2	4.447
3	6.333
4	7.197
5	8.114
6	8.980

7	9.865
8	10.775
17	18.698
18	19.599
19	20.497
20	21.348
21	22.264
22	23.137
23	24.006
24	24.915

$f = 244.73$ KC

$T = 26.8^{\circ}C$

$p = 1200.8$ mm

$E_D^A = 0.93$ volts

$$8\lambda = 14.1517 \text{ mm}$$

$$\lambda = 1.76896 \text{ mm}$$

$$V = 432.918 \text{ meters/sec}$$

For crystal-set D, readings were taken for each peak from the 1st peak to the 9th peak and from the 19th peak to the 27th peak.

6.

Peak	Spacing (mm)
1	5.094
2	5.720
3	6.359
4	6.982
5	7.619
6	8.240
7	8.870
8	9.507
9	10.119
19	16.432
20	17.031
21	17.685
22	18.291
23	18.939
24	19.549
25	20.179
26	20.820
27	21.430

$f = 343.65$ KC

$T = 25.7^{\circ}C$

$p = 1200.0$ mm

$E_D^A = 0.94$ volts

Subtracting the 1st peak from the 19th peak, etc., and averaging gives

$$9\lambda = 11.3162 \text{ mm}$$

$$\lambda = 1.25736 \text{ mm}$$

$$V = 432.092 \text{ meters/sec}$$

9.

Peak	Spacing (mm)
1	5.092
2	5.716
3	6.352
4	6.984
5	7.618
6	8.235
7	8.860
8	9.504
9	10.113
19	16.431
20	17.022
21	17.676
22	18.280
23	18.931
24	19.545
25	20.169
26	20.820
27	21.415

f = 343.65 KC

T = 25.6°C

p = 1200.0 mm

E_0^A = 0.94 volts

9λ = 11.3127 mm

λ = 1.25697 mm

V = 431.958 meters/sec

10.

Peak	Spacing (mm)
1	5.095
2	5.712
3	6.359
4	6.983
5	7.612
6	8.239
7	8.861
8	9.512
9	10.115
19	16.429
20	17.021
21	17.679
22	18.279
23	18.930
24	19.552
25	20.165
26	20.820
27	21.415

f = 343.65 KC

T = 26.0°C

p = 1199.9 mm

E_0^A = 0.94 volts

$$9\lambda = 11.3113 \text{ mm}$$

$$\lambda = 1.25681 \text{ mm}$$

$$V = 431.903 \text{ meters/sec}$$

11.

Peak	Spacing (mm)
1	5.100
2	5.712
3	6.352
4	6.986
5	7.611
6	8.241
7	8.858
8	9.509
9	10.106
19	16.431
20	17.021
21	17.681
22	18.278
23	18.924
24	19.549
25	20.168
26	20.816
27	21.409

$f = 343.65$ KC

$T = 26.1\,^{\circ}C$

$p = 1199.9$ mm

$E_D^A = 0.94$ volts

$$9\lambda = 11.3113 \text{ mm}$$

$$\lambda = 1.25681 \text{ mm}$$

$$V = 431.903 \text{ meters/sec}$$

For crystal-set E, readings were taken for 5-peak intervals.

12.

Peak	Spacing (mm)
1	5.706
6	8.246
11	10.769
16	13.280
21	15.786
26	18.317
31	20.846
36	23.373

$f = 428.77$ KC

$T = 26.7\,^{\circ}C$

$p = 1199.7$ mm

$E_D^A = 0.95$ volts

Subtracting the 1st peak from the 21st peak, etc., and averaging gives

$$10\lambda = 10.0803 \text{ mm}$$

$$\lambda = 1.00803 \text{ mm}$$

$$V = 432.213 \text{ meters/sec}$$

13.

Peak	Spacing (mm)
1	5.707
6	8.242
11	10.770
16	13.278
21	15.786
26	18.316
31	20.848
36	23.372

$f = 428.78$ KC

$T = 26.7^\circ C$

$p = 1200.2$ mm

$E_D^A = 0.95$ volts

$$10\lambda = 10.0813 \text{ mm}$$

$$\lambda = 1.00813 \text{ mm}$$

$$V = 432.266 \text{ meters/sec}$$

14.

Peak	Spacing (mm)
1	5.703
6	8.242
11	10.769
16	13.279
21	15.788
26	18.313
31	20.845
36	23.372

$f = 428.78$ KC

$T = 26.7^\circ C$

$p = 1200.7$ mm

$E_D^A = 0.95$ volts

$$10\lambda = 10.0813 \text{ mm}$$

$$\lambda = 1.00813 \text{ mm}$$

$$V = 432.266 \text{ meters/sec}$$

For crystal-set F, readings were taken for 4-peak intervals.

15.

Peak	Spacing (mm)
1	4.180
5	5.611
9	7.021
13	8.439
17	9.858
21	11.270
25	12.687
29	14.104
33	15.525
37	16.939
41	18.358
45	19.772
49	21.190
53	22.602
57	24.019
61	25.435

f = 609.18 KC

T = 26.0 °C

p = 1200.4 mm

$\hat{E}_D$ = 0.93 volts

Subtracting the 1st peak from the 33rd peak, etc., and averaging gives

$$16\lambda = 11.3334 \text{ mm}$$

$$\lambda = 0.70834 \text{ mm}$$

$$V = 431.507 \text{ meters/sec}$$

16.

Peak	Spacing (mm)
1	4.184
5	5.601
9	7.024
13	8.442
17	9.856
21	11.270
25	12.688
29	14.101
33	15.525
37	16.939
41	18.353
45	19.771
49	21.189
53	22.600
57	24.020
61	25.437

f = 609.18 KC

T = 26.0 °C

p = 1200.1 mm

$\hat{E}_D$ = 0.93 volts

$$16\lambda = 11.3335 \text{ mm}$$

$$\lambda = 0.70834 \text{ mm}$$

$$V = 431.507 \text{ meters/sec}$$

17.

Peak	Spacing (mm)
1	4.182
5	5.606
9	7.021
13	8.443
17	9.855
21	11.277
25	12.690
29	14.111
33	15.531
37	16.946
41	18.360
45	19.778
49	21.190
53	22.607
57	24.028
61	25.444

$f = 609.18$ KC

$T = 26.1^{\circ}$ C

$p = 1200.5$ mm

$E_D^A = 0.93$ volts

$$16\lambda = 11.3374 \text{ mm}$$

$$\lambda = 0.70859 \text{ mm}$$

$$V = 431.659 \text{ meters/sec}$$

18.

Peak	Spacing (mm)
1	4.182
5	5.604
9	7.025
13	8.443
17	9.855
21	11.271
25	12.693
29	14.111
33	15.530
37	16.943
41	18.356
45	19.777
49	21.190
53	22.611
57	24.023
61	25.447

$f = 609.18$ KC

$T = 26.1^{\circ}$ C

$p = 1200.0$ mm

$E_D^A = 0.93$ volts

$$16\lambda = 11.3366 \text{ mm}$$

$$\lambda = 0.70854 \text{ mm}$$

$$V = 431.628 \text{ meters/sec}$$

For crystal-set G, readings were taken for 8-peak intervals.

19.

Peak	Spacing (mm)
1	2.368
9	3.791
17	5.220
25	6.641
33	8.067
41	9.498
49	10.922
57	12.347
65	13.771
73	15.192
81	16.621
89	18.047
97	19.471
105	20.895
113	22.320
121	23.746

$f = 1209.75$ KC

$T = 25.4^{\circ}$C

$p = 1199.5$ mm

$E_D^A = 0.94$ volts

Subtracting the 1st peak from the 65th peak, etc., and averaging gives

$$32\lambda = 11.4011 \text{ mm}$$

$$\lambda = 0.35628 \text{ mm}$$

$$V = 431.009 \text{ meters/sec}$$

20.

Peak	Spacing (mm)
1	2.365
9	3.791
17	5.219
25	6.640
33	8.068
41	9.498
49	10.922
57	12.342
65	13.766
73	15.192
81	16.619
89	18.044

$f = 1209.78$ KC

$T = 25.3^{\circ}$C

$p = 1200.0$ mm

$E_D^A = 0.90$ volts

97	19.470
105	20.890
113	22.315
121	23.741

$$32\lambda = 11.3990 \text{ mm}$$

$$\lambda = 0.35622 \text{ mm}$$

$$V = 430.947 \text{ meters/sec}$$

21. Peak Spacing (mm)

Peak	Spacing (mm)
1	2.363
9	3.790
17	5.217
25	6.640
33	8.068
41	9.495
49	10.920
57	12.341
65	13.764
73	15.191
81	16.615
89	18.040
97	19.463
105	20.890
113	22.312
121	23.739

$f = 1209.69$ KC

$T = 25.2^\circ$ C

$p = 1200.2$ mm

$E_D^A = 0.90$ volts

$$32\lambda = 11.3975 \text{ mm}$$

$$\lambda = 0.35617 \text{ mm}$$

$$V = 430.855 \text{ meters/sec}$$

It was thought desirable to find out to what degree of accuracy the measurements could be repeated. For this reason the following set of data was taken and may be compared with data sets 8, 9, 10, and 11. The period between the two measurements was approximately three weeks and during that time measurements were made with other crystals.

22. Peak	Spacing (mm)
1	5.070
2	5.689
3	6.341
4	6.945
5	7.573
6	8.188
7	8.798
8	9.451
9	10.061
19	16.361
20	16.973
21	17.616
22	18.235
23	18.866
24	19.500
25	20.118
26	20.763
27	21.372

$f = 343.65$ KC

$T = 26.2\,^\circ C$

$p = 1200.2$ mm

$\hat{E}_D = 0.91$ volts

$9\lambda = 11.3137$ mm

$\lambda = 0.12571$ mm

$V = 432.002$ meters/sec

In order to determine whether or not the variation in the velocity of sound in ammonia was due to the interferometer, the following eight sets of data (at four different frequencies) were taken using argon as the medium.

Using crystal-set B, readings were taken for each peak from the 1^{st} peak to the 8^{th} peak and from the 18^{th} peak to the 25^{th} peak.

23. Peak	Spacing (mm)
1	5.440
2	6.078
3	6.767
4	7.405
5	8.096
6	8.728

7	9.424	f = 244.69 KC
8	10.052	T = 28.1°C
18	16.692	p = 1200.2 mm
19	17.333	E_D^A = 0.98 volts
20	18.020	
21	18.656	
22	19.347	
23	19.979	
24	20.676	
25	21.304	

Subtracting the 1st peak from the 18th peak, etc., and averaging gives

$$8.5\lambda = 11.2521 \text{ mm}$$

$$\lambda = 1.32378 \text{ mm}$$

$$V = 323.916 \text{ meters/sec}$$

24.

Peak	Spacing (mm)	
1	5.441	f = 244.69 KC
2	6.081	
3	6.770	T = 28.2°C
4	7.404	
5	8.097	p = 1200.2 mm
6	8.729	
7	9.426	E_D^A = 0.98 volts
8	10.056	
18	16.697	
19	17.340	
20	18.029	
21	18.655	
22	19.352	
23	19.984	
24	20.675	
25	21.309	

$$8.5\lambda = 11.2546 \text{ mm}$$

$$\lambda = 1.32407 \text{ mm}$$

$$V = 323.987 \text{ meters/sec}$$

Using crystal-set C, readings were taken for each peak from the 1st peak to the 8th peak and from the 25th peak to the 32nd peak.

25.

Peak	Spacing (mm)
1	5.500
2	6.005
3	6.562
4	7.066
5	7.594
6	8.140
7	8.633
8	9.184
25	18.100
26	18.597
27	19.151
28	19.653
29	20.178
30	20.725
31	21.221
32	21.769

f = 307.68 KC

T = 26.7° C

p = 1200.1 mm

E_D^A = 0.92 volts

Subtracting the 1st peak from the 25th peak, etc., and averaging gives

$$12\lambda = 12.5888 \text{ mm}$$

$$\lambda = 1.04907 \text{ mm}$$

$$V = 322.778 \text{ meters/sec}$$

26.

Peak	Spacing (mm)
1	5.499
2	6.003
3	6.561
4	7.067
5	7.593
6	8.134
7	8.632
8	9.186
25	18.102
26	18.601
27	19.158
28	19.659
29	20.181

f = 307.68 KC

T = 26.8° C

p = 1200.0 mm

E_D^A = 0.92 volts

30	20.731
31	21.222
32	21.771

$$12\lambda = 12.5938 \text{ mm}$$

$$\lambda = 1.04948 \text{ mm}$$

$$V = 322.904 \text{ meters/sec}$$

Using crystal-set E_g readings were taken for 4-peak intervals.

27.

Peak	Spacing (mm)
1	5.811
5	7.315
9	8.819
13	10.324
17	11.831
21	13.344
25	14.849
29	16.354
33	17.856
37	19.354
41	20.854
45	22.352
49	23.850
53	25.347

$f = 428.75$ KC

$T = 25.9^\circ$ C

$p = 1200.0$ mm

$E_D^A = 0.96$ volts

Subtracting the 1st peak from the 29th peak, etc., and averaging gives

$$14\lambda = 10.5249 \text{ mm}$$

$$\lambda = 0.75178 \text{ mm}$$

$$V = 322.326 \text{ meters/sec}$$

28.

Peak	Spacing (mm)
1	5.811
5	7.318
9	8.820
13	10.328
17	11.836
21	13.347
25	14.852
29	16.359

$f = 428.75$ KC

$T = 26.0^\circ$ C

$p = 1200.6$ mm

$E_D^A = 0.96$ volts

33	17.860
37	19.361
41	20.856
45	22.360
49	23.856
53	25.355

$$14\lambda = 10.5279 \text{ mm}$$

$$\lambda = 0.75199 \text{ mm}$$

$$V = 322.416 \text{ meters/sec}$$

Using crystal-set G, readings were taken for 11-peak intervals.

29.

Peak	Spacing (mm)
1	1.716
12	3.182
23	4.651
34	6.121
45	7.591
56	9.063
67	10.536
78	12.007
89	13.477
100	14.942
111	16.407
122	17.872
133	19.336
144	20.801
155	22.266
166	23.726

$f = 1209.28$ KC

$T = 26.0\,^{\circ}C$

$p = 1199.6$ mm

$E_D^A = 0.95$ volts

Subtracting the 1st peak from the 89th peak, etc., and averaging gives

$$44\lambda = 11.7450 \text{ mm}$$

$$\lambda = 0.26693 \text{ mm}$$

$$V = 322.793 \text{ meters/sec}$$

30.

Peak	Spacing (mm)
1	1.714
12	3.181
23	4.649
34	6.119
45	7.590
56	9.061
67	10.532
78	12.007
89	13.474
100	14.942
111	16.404
122	17.870
133	19.337
144	20.796
155	22.261
166	23.723

f = 1209.22 KC

T = 26.0°C

p = 1199.7 mm

E_D^A = 0.96 volts

44λ = 11.7443 mm

λ = 0.26692 mm

V = 322.765 meters/sec

The calculations are collected here for convenience. The ammonia data is corrected to 26°C. The corrections are made on the basis of the perfect gas law, i.e., the velocity is proportional to the square root of the absolute temperature. This is justified since the corrections are for very small temperature differences. The argon data are corrected to 0°C and may be compared with the value given in the International Critical Tables, i.e., 307.8 meters/sec.

Ammonia Data

Data Set	f (KC)	$\log(\frac{\omega}{p})$	V(uncor.) (m/sec)	T (°K)	V (m/sec)	V(av.) (m/sec)	$V^2 X10^{-5}$ (m/sec)	$V^2 X10^{-5}$(av.) (m/sec)
1	191.45	3.00	431.750	298.6	432.18	432.15	1.868	1.868
2	"	"	431.784	298.8	432.07		1.867	
3	"	"	431.872	298.8	432.16		1.868	
4	"	"	431.876	298.8	432.16		1.868	
5	244.73	3.11	432.604	299.5	432.39	432.36	1.870	1.869
6	"	"	432.805	299.8	432.37		1.870	
7	"	"	432.918	300.0	432.36		1.869	
8	343.65	3.25	432.092	298.9	432.31	432.03	1.869	1.866
9	"	"	431.958	298.8	432.25		1.868	
10	"	"	431.903	299.2	431.90		1.865	
11	"	"	431.903	299.3	431.83		1.865	
22	"	"	432.002	299.4	431.87		1.865	
12	428.77	3.35	432.213	299.9	431.69	431.73	1.865	1.864
13	428.78	"	432.226	299.9	431.75		1.865	
14	"	"	432.266	299.9	431.75		1.865	
15	609.18	3.50	431.507	299.2	431.51	431.53	1.862	1.862
16	"	"	431.507	299.2	431.51		1.862	
17	"	"	431.659	299.3	431.59		1.863	
18	"	"	431.628	299.3	431.56		1.862	
19	1209.75	3.80	431.009	298.6	431.44	431.43	1.861	1.861
20	1209.78	"	430.947	298.5	431.43		1.861	
21	1209.69	"	430.855	298.4	431.43		1.861	

Argon Data

Data Set	f (KC)	$\log(\frac{\omega}{p})$	V(uncor.) (m/sec)	T (°K)	V (m/sec)	V(av.) (m/sec)
23	244.69	3.11	323.916	301.3	308.50	308.50
24	"	"	323.987	301.2	308.50	
25	307.68	3.21	322.778	299.9	308.06	308.10
26	"	"	322.904	300.0	308.15	
27	428.75	3.35	322.326	299.1	308.05	308.07
28	"	"	322.416	299.2	308.10	
29	1209.28	3.80	322.793	299.2	308.46	308.44
30	1209.22	"	322.765	299.2	308.43	

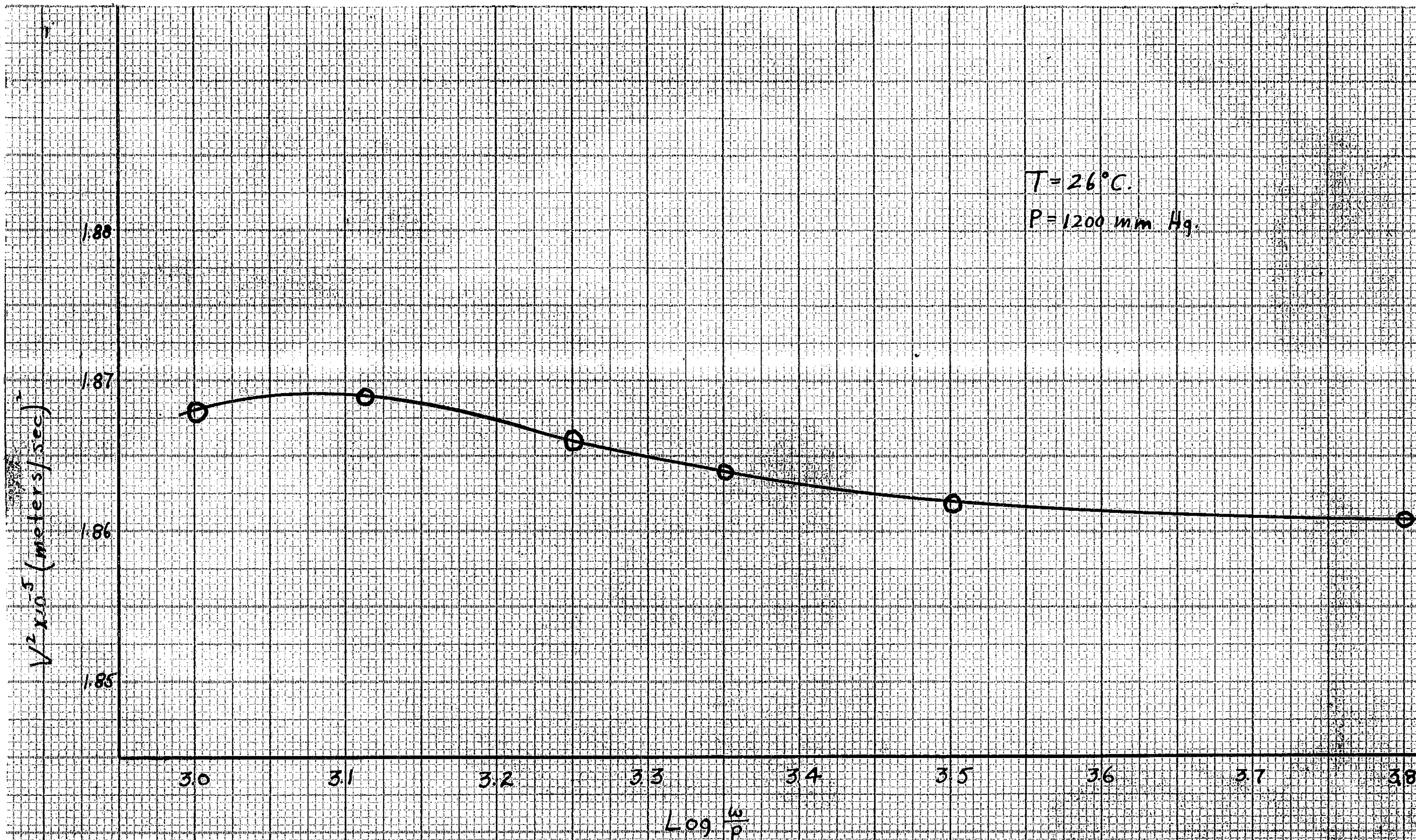
VI

Discussion of Results

The results of twenty-two data runs on ammonia at six different frequencies are shown plotted in Fig. 3. These data were taken at temperatures very close to 26°C. and corrected to that value. The pressures were all very nearly 1200 mm. Hg. The velocity of sound in ammonia appears to have a maximum at about 240 KC after which it decreases slightly with frequency.

The results of eight data runs on argon at four different frequencies show a variation of approximately 0.4 meters/sec for the velocity of sound. This variation is 0.13% of the value given in the International Critical Tables. It is reasonable to assume that the velocity of sound in argon does not vary with frequency. Therefore, if this is taken to be the limit of accuracy of the experiments, then the overall uncertainty in the measurements is of the order of one part in eight hundred.

A discussion of the measurements of the independent factors such as wavelength, frequency, etc., is in order. The uncertainty in locating a peak is taken as 0.003 mm. By the manner in which the data were taken and the calculations made for the wavelength, the maximum uncertainty is of the order of 0.0001 mm. in 10 mm. or approximately one part in ten thousand. By this method of computation the maximum uncertainty occurs for the data of the lowest frequency crystals. The uncertainty in the frequency is of the order of one part in twenty thousand. The discussion of Sec. II indicates that the experimental variation of the temperature



The Velocity of Sound in Ammonia
as a Function of Frequency

Figure 3

and pressure should have a negligible effect upon the velocity of sound.

The ammonia used in all the experiments was from one cylinder of anhydrous ammonia. A chemical analysis indicated that it was 98.4% pure with slight traces of water vapor and no detectable CO_2.

VII

Conclusions

The work presented in this paper is thought to be only a prelude to the work that can be done on polyatomic gases with the double-crystal acoustic interferometer. It is felt that the experimental results reported here thoroughly demonstrate the utility of this kind of acoustic interferometer as an accurate measuring device.

To the degree of accuracy at which the velocity of sound in ammonia was determined, it must be concluded that the existence of a measurable velocity dispersion, in the region of investigation, is doubtful. Other workers[7,8] in the field have reported velocity dispersion ranging in magnitude from 2 to 4 meters/sec. Their measurements were made by varying the pressure. The discussion of the pressure dependence of the velocity of sound in ammonia indicates that their measurements do not conclusively demonstrate the existence of dispersion.

VIII

Appendix

A. Calculation of the Error in Measurement of the Wavelength in Considering the Standing Wave Peaks to be Spaced Apart

From the interferometer equation[1]

$$1 \qquad \left|\frac{E_R^B}{E_D^A}\right| = \frac{|C''|}{\left[\cosh^2\left(\alpha\frac{L}{\lambda}+b'\right) - \cos^2\left(2\pi\frac{L}{\lambda}+a'\right)\right]^{3/2}}$$

the location of the peaks can be found by setting the derivative equal to zero. (It is assumed that the driving voltage $|E_D^A|$ remains constant.)

$$2 \qquad \frac{d\left|\frac{E_R^B}{E_D^A}\right|}{d\left(\frac{L}{\lambda}\right)} = \frac{-|C''|\cdot\frac{1}{2}A}{\left[\cosh^2\left(\alpha\frac{L}{\lambda}+b'\right) - \cos^2\left(2\pi\frac{L}{\lambda}+a'\right)\right]^{3/2}}$$

where

$$3 \qquad A = 2\alpha\cosh\left(\alpha\frac{L}{\lambda}+b'\right)\sinh\left(\alpha\frac{L}{\lambda}+b'\right) + 4\pi\cos\left(2\pi\frac{L}{\lambda}+a'\right)\sin\left(2\pi\frac{L}{\lambda}+a'\right)$$

At a peak

$$4 \qquad \alpha\sinh\left[2\left(\alpha\frac{L}{\lambda}+b'\right)\right] + 2\pi\sin\left[2\left(2\pi\frac{L}{\lambda}+a'\right)\right] = 0$$

If $\frac{L_1}{\lambda}$ is the location of a peak, then $\frac{L_1}{\lambda}+\frac{n}{2}+\epsilon$ is the location of another. Therefore

$$5 \qquad \alpha\sinh\left[2\left(\alpha\frac{L_1}{\lambda}+b'\right)\right] + 2\pi\sin\left[2\left(2\pi\frac{L_1}{\lambda}+a'\right)\right] = 0$$

$$6 \quad \alpha \sinh\left[2\left\{\alpha\left(\frac{L_1}{\lambda}+\frac{n}{2}+\epsilon\right)+b'\right\}\right]+2\pi \sin\left[2\left\{2\pi\left(\frac{L_1}{\lambda}+\frac{n}{2}+\epsilon\right)+a'\right\}\right]=0$$

Rewriting eq.6 gives

$$7 \quad \alpha\left[\sinh 2\left(\alpha\frac{L_1}{\lambda}+b'\right)\cosh 2\alpha\left(\frac{n}{2}+\epsilon\right)+\cosh 2\left(\alpha\frac{L_1}{\lambda}+b'\right)\sinh 2\alpha\left(\frac{n}{2}+\epsilon\right)\right]$$
$$+2\pi\left[\sin 2\left(2\pi\frac{L_1}{\lambda}+a'\right)\cos 4\pi\left(\frac{n}{2}+\epsilon\right)+\cos 2\left(2\pi\frac{L_1}{\lambda}+a'\right)\sin 4\pi\left(\frac{n}{2}+\epsilon\right)\right]=0$$

Assume now that

$$\cosh\left(\frac{n}{2}+\epsilon\right)\sim 1$$

$$\cos 4\pi\left(\frac{n}{2}+\epsilon\right)\sim 1$$

$$\sinh 2\alpha\left(\frac{n}{2}+\epsilon\right)\sim 2\alpha\left(\frac{n}{2}+\epsilon\right)$$

$$\sin 4\pi\left(\frac{n}{2}+\epsilon\right)\sim 4\pi\epsilon$$

With these assumptions eq.7 becomes

$$2\alpha^2\left(\frac{n}{2}+\epsilon\right)\cosh 2\left(\alpha\frac{L_1}{\lambda}+b'\right)+8\pi^2\epsilon\cos 2\left(2\pi\frac{L_1}{\lambda}+a'\right)=0$$

and solving for ϵ yields

$$\epsilon=\frac{-\alpha^2 n\cosh 2\left(\alpha\frac{L_1}{\lambda}+b'\right)}{2\alpha^2\cosh 2\left(\alpha\frac{L_1}{\lambda}+b'\right)+8\pi^2\cos 2\left(2\pi\frac{L_1}{\lambda}+a'\right)}$$

Since

$$\cos 2\left(2\pi\frac{L_1}{\lambda}+a'\right)=2\cos^2\left(2\pi\frac{L_1}{\lambda}+a'\right)-1$$

and near a peak

$$\cos^2\left(2\pi\frac{L_1}{\lambda}+a'\right)\sim 1$$

ϵ becomes

$$\epsilon = \frac{-\alpha^2 n \cosh 2\left(\alpha \frac{L_1}{\lambda} + b'\right)}{2\alpha^2 \cosh 2\left(\alpha \frac{L_1}{\lambda} + b'\right) + 8\pi^2}$$

To get an idea of the magnitude of this error let

$\alpha = 0.02$ (approximate for ammonia)

$\frac{L_1}{\lambda} = 100$

$n = 25$ wavelengths

$b' = 0.04$

and

$\epsilon = -0.0037$ wavelengths.

Thus the error in assuming that the peaks are spaced a half wavelength apart, when the 200th and 250th peaks are considered, is negligible compared to the wavelength measured in the experimental work presented here.

B. List of Symbols

a - constant of the cohesive pressure term in the equation of state

a' - interferometer constant

b' - interferometer constant

C'' - interferometer constant

C_p, C_v - respectively the molar heat capacities at constant pressure and constant volume

C_p^0, C_v^0 - C_p, C_v at low frequencies

C_p^∞, C_v^∞ - C_p, C_v at high frequencies

E_A^D, E_R^B - respectively the voltage at the driver and receiver crystals

E_A, E_T - respectively the adiabatic and isothermal elasticities

f - frequency

g - constant

k - wave vector

k_1 - real component of k

k_2 - imaginary component of k $(k_2 = \alpha)$

L - spacing distance between crystals

ℓ - constant of the cohesive pressure term in the equation of state

M - molecular weight

n - number of wavelengths

P - excess pressure caused by sound wave

p - total pressure

R - gas constant

T - Kelvin temperature

t - time

V - velocity of sound

v - specific volume

X - distance in direction of propagation

α - absorption coefficient

γ - ratio of heat capacities $= \frac{C_p}{C_v}$

δ - correction term of the volume in the equation of state

ϵ - error term

λ - wavelength

ρ - density

τ - relaxation time constant

ω - angular frequency $= 2\pi f$

IX

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